

THE WEB-SYNDIC SYSTEM

USER'S MANUAL

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1 Overview

Web-SynDic stands for **w**eb-based demonstration and test of the **s**yntactic algorithms for solving linear **D**iophantine equations in nonnegative integers. These algorithms, developed at CS Department of PetrSU, are a novel type of algorithms for efficient solving some classes of nonnegative linear Diophantine equations (NLDE or LDE in nonnegative integers) by syntactic (parsing) methods.

These syntactic algorithms seem to be promising tool for solving some classes of NLDE system; more exactly a class of NLDE system, associated with formal grammars (ANLDE systems). For this class the syntactic algorithms allow efficient (polynomial and pseudo-polynomial) computations comparing with the general NLDE case when the same computational problems are NP-complete or even overNP [6].

A user just sends a test ANLDE system to Web-SynDic; the latter responds with the solution and some characteristics of the computation. This allows to present the key features of the syntactic algorithms, test them, estimate the efficiency, etc.

A user is assumed to be a researcher in Diophantine analysis, formal grammars, integer programming, and related fields. She/He has an access to the Internet via a standard browser.

Note, that the web system does not allow a user to have a direct access to the demonstrated&tested algorithms but shows only the outcome of their work. Detailed information about the syntactic algorithms and ANLDE theory can be found in [1, 2, 3, 4].

2 Description

2.1 Conventions

In this document the following conventions for fonts are used.

Typewriter font URL address, names of programs and utilities, input text for a web form.

Bold font 1) Name of buttons available in Web-SynDic pages. 2) Important notes on using Web-SynDic.

San-serif font Name of forms and text areas available in Web-SynDic pages.

Underlined text Names of links available in Web-SynDic pages.

2.2 Abbreviations and terms

Term	Description
ANLDE system	Associated with a formal grammar, NLDE system. See [2, 3].
CF-grammar	Context-free grammar
CS	Computer Science
CSDept	Computer Science Department. The PetrSU CSDept web-site is http://www.cs.karelia.ru
Hilbert basis	A set of all indecomposable (minimal) solutions of a homogeneous NLDE system.
ILP	Integer linear programming
Indecomposable solution	A particular solution that is not a sum of two particular solutions.
lp.solve	The non-commercial linear programming code, written in ANSI C by Michel Berkelaar. Also it supports ILP problems. Available on http://www.cs.sunysb.edu/~algorithm/implement/lpsolve/implement.shtml
NLDE	Nonnegative linear Diophantine equations, i.e. their solutions are in nonnegative integers and coefficients are integer. See for example [6, 7, 8].
Particular solution	Any non-trivial solution of a homogenous NLDE system.
PetrSU	Petrozavodsk State University, http://petrsu.karelia.ru
Session	Web-SynDic logical session for continuous time period of user working with the Web-SynDic server.
Slopes	Algorithm of M.Filgueiras and A.-P.Tomás for searching Hilbert basis of a homogenous NLDE system, available on http://www.ncc.up.pt/~apt/dioph/
Syntactic Algorithms	The algorithms that solve ANLDE system by constructing some derivations in the corresponding formal grammar, see [4, 3]. Web-SynDic is intended to demonstrate and test such algorithms.
Trivial solution	All-zero solution $\mathbb{O} = (0, \dots, 0)$ of a homogeneous NLDE system.
Web-SynDic	It stands for Web -based demonstrating and testing the Syntactic algorithms for solving nonnegative linear Diophantine equations.

3 Getting Started

3.1 What is Web-SynDic

Welcome to Web-SynDic — a web system for demonstrating and testing novel syntactic method for searching nonnegative integer solutions of Linear Diophantine Equations (LDE in nonnegative integers or Nonnegative LDE or NLDE for short).

Web-SynDic works with a special class of homogenous NLDE systems that are Associated with context-free grammars (ANLDE systems for short).

The word SynDic has the roots in theory of formal grammars and Diophantine analysis:

starting from the word **Syntactic** one can derive the word SynDic applying the grammar rule $tact \rightarrow D$.

where **D** means **Diophant**.

Thus, the name Web-SynDic tactfully reflects a symbiosis of Syntactic and Diophantine concepts inside a Web environment.

3.2 Initial idea and example

NLDE system is a system of linear equations with integer coefficients and with solutions in nonnegative integers. A homogenous ANLDE system is a particular case of NLDE one:

1. Each unknown must appear at most one time on all left-hand sides of the equations.
2. On the right-hand sides of the equations an unknown may appear arbitrary but with only with nonnegative coefficients.

The general form (n is number of equations, m is number of unknowns):

$$\left\{ \begin{array}{l} \sum_{k=1}^n x_i = \sum_{i=1}^m a_{2i} x_i \\ \sum_{k=2}^n x_i = \sum_{i=1}^m a_{2i} x_i \\ \dots \\ \sum_{k=n}^n x_i = \sum_{i=1}^m a_{ni} x_i \end{array} \right.$$

Here is an example of a typical ANLDE system (in traditional mathematical notation for text representation of math formula):

$$\begin{array}{l} x1 + x2 = 2*x1 + 3*x3 \\ x3 + x4 = x1 + 2*x2 + x3 \end{array}$$

You can process it with [Process ANLDE system](#) link in the menu on the left of the main page of Web-SynDic. Then you see Hilbert basis (minimal solutions) of this ANLDE system, found by the syntactic algorithm:

$$h1 = (1,1,0,3) \text{ and } h2 = (0,3,1,6)$$

The same result is given by the slopes algorithm (M.Filgueiras and A.-P.Tomas); this is an alternative solver, supported by Web-SynDic. Select slopes as alternative solver before solving the ANLDE system and then compare the results.

Thus, any solution of this ANLDE system has the form:

$$x = a*h1 + b*h2 \text{ for nonnegative integer } a \text{ and } b$$

Using the cost function

$$x1 + x2 + x3 + x4 \rightarrow \min$$

one can solve the optimization problem with this cost function and the considered ANLDE system as constraints and get one particular solution of the ANLDE system by standard integer programming methods:

$$x = (1,1,0,3)$$

The optimal solution is always equal to a Hilbert basis element.

As ILP solver Web-SynDic supports `lp_solve` (M.Berkelaar). This solver is based on the well-known technique of simplex algorithm among with branch-and-bound method. For the cost function the sum of all unknowns with unit coefficients are used.

3.3 Features of the Web-SynDic system

Using Web-SynDic you can test our syntactic algorithm on homogenous ANLDE systems, look at its efficiency, compare it with available alternative algorithms. Use the menu on the left of the main page to access any of these features.

The basic Web-SynDic feature is **processing** a single ANLDE system or a set of ANLDE systems. Constraints on CPU time of this processing and on dimensions of ANLDE systems are controlled with **user limits**. Your opinion about Web-SynDic can be given with **send notes** feature.

Starting Web-SynDic you are working as a **guest**. To access more function, you should **register** and **log in**. During the guest session you can change the user limits, but the changes are lost whenever the session is over. The lifetime of your session is not limited, however the session is terminated after idle period of 15 min.

3.4 Solving a test ANLDE system

The basic Web-SynDic functionality includes solving a single test ANLDE system. Solution of the following example system

$$\begin{cases} x_1 + x_4 = 2x_1 + 3x_3 \\ x_2 + x_3 = x_1 + 2x_2 + x_3 \end{cases}$$

can be done as described below.

To receive form to enter the ANLDE system, user should click on the [Process an ANLDE System](#) link in Main Menu.

All ANLDE systems in the Web-SynDic system are represented in traditional mathematical style. The mentioned system will look like

$$\begin{aligned} x1 + x4 &= 2*x1 + 3*x3 \\ x2 + x3 &= x1 + 2*x2 + x3 \end{aligned}$$

Type this text into the **System** text field. After that click on **Solve** button. Web-SynDic solves the test system using the syntactic algorithms and returns the report on solution to the user.

3.5 Generating a test ANLDE system

Web-SynDic allows to generate ANLDE systems instead of manual input. Such a generated system characterized with simple Hilber basis. The reason is that the generation is based on original methods by Kirill Kulakov [5]. These methods were also used for comprehensive automatic testing of the syntactic solver.

To force Web-SynDic for the ANLDE generation, click on the **Generate** button in the corresponding form. After that the same form is returned with containing the generated ANLDE system in the **System** field.

To save an ANLDE system, shown in **System** text field, click on the **Save** button, You will see a new browser window containing the ANLDE system. Then you can save it in a text file using standard browser features on file saving like “Save As...” item of “File” menu.

4 Reference Manual

4.1 User Session

One of the main concept of the Web-SynDic system is session. There are two types of sessions: a physical server session and a logical Web-SynDic session. Web-SynDic session stores all information about current work in the system, for example ANLDE systems, user profile etc.

Web-SynDic session is a continuous time period of user’s working with the Web-SynDic. Further we will use term “session” instead of “Web-SynDic session”.

Session is established when any user (regular on registered) start to use the system. It is open while a user logs out or closes the connection. If a user logs out, then the session is closed at once, but if a user closes the connection, for example closes the browser window, the session is still alive for a some period of time (default is 15 min and configured by Jakarta Tomcat server).

If a user has established a session and does not use the system with a large time period the session will be terminated automatically. For using the web-system a user should establish a session again.

Each session has own identifier on the server side. A new identifier of session is generated whenever the session has been established.

If cookies are enabled in user’s browser, then the identifier of server session is passed by cookies. If they are disabled the session identifier is passed by the URL string in a browser window.

4.2 Registration and Log In

There are two general types of users: a regular user and a registered user.

A regular user is any Web-SynDic user and she/he is not required to register and log in the web system. A registered user has to have an account in the web system and his/her personal information and limits for solving and generating ANLDE systems are saved in personal user profile.

Web-SynDic allows any regular user to register whenever she/he wishes. When the registration is complete, this user may log in the web system (using assigned unique identifier: nick name and password), afterwards some additional Web-SynDic features may be used. Web-SynDic allows a registered user to edit and store personal profile with user limits independently on session lifetime.

The registration is initialized with the **Register** button in the **Log In** form. Afterward the web system returns the form, shown in Fig. 1. The user feeds the fields of the form with information, he/she wishes to provide about himself/herself, and clicks on the **Register** button to complete registration. The required fields for filling are marked with “*” sign. A nickname may contain only latin letters, numbers and underscores; also a nickname contains at least 2 and no more than 32 characters. A password may contain at least 4 and no more than 32 characters. If there is an error in filling fields, then Web-SynDic responds with a corresponding error message; it includes a reason of the error.

When the registration has been completed, the user may log in to the Web-SynDic system. To do this, he/she enters login and password in the **Log In** form and clicks on the **Log in** button.

Figure 1: Registration

After successful login, contents of main window is changed. Log In area now contains user status in the Web-SynDic system. “User Profile” link appears in User Menu to provide additional Web-SynDic feature. After clicking “User profile” link the user information form will be shown, see Fig. 2. A user can change her/his information, such as full name, email, personal information. Also he/she can change password or remove account. A user selects corresponding checkboxes, change information and press ‘Submit’ button. New changes will be saved in the user profile.

Logout from the web system can be performed using Log out button. It appears after successful login in Log In area. It is strongly recommended for every registered user to explicitly log out Web-SynDic at the end of her/his session.

4.3 Processing a test ANLDE system

Web-SynDic can be used for solving a test ANLDE system and shows a user the report on solution. An ANLDE system should be given manually by a user or generated automatically by a selected generator. Recently, only homogeneous ANLDE systems are supported by Web-SynDic.

To input an ANLDE system a user clicks on the [ANLDE System](#) link, whereupon he/she

Figure 2: User information

will be returned with the corresponding form, see Fig. 3. Then the user should input her/his ANLDE system in traditional mathematical notation, for example:

$$\begin{aligned}x1 + x2 &= 2*x2 + 3*x3 \\x4 &= x1 + 2*x2\end{aligned}$$

Web-SynDic provides two ways to perform this:

- Manual input to the Input a test ANLDE system (or simply System) text area of the form. This can be used if a user has his/her own system.
- Otherwise, Web-SynDic can generate an ANLDE system and insert it into the System text area of the form. The web system uses user limits as input parameters of the generation process.

To generate a test ANLDE system, a user should click on the **Generate** button of the form, after that the web system executes the sequence of operations described above.

To save current ANLDE system, i.e. the system shown in System text field, the **Save** button of the form might be used. The web system opens window containing the same ANLDE system.

Figure 3: Process ANLDE System

Actual saving of the system on a user's local storage can be performed using standard browser features on file saving.

For solving a test ANLDE system, a user clicks on the **Solve** button of the form. Web-SynDic solves the test system using the syntactic algorithms and returns the report on solution to the user.

Report on solution provides the following data:

1. The given test ANLDE system in mathematical notation.
2. Different metrics of solution process (solver efficiency estimates).
3. Solving machine characteristics.
4. Basis solutions of the given ANLDE system.

The Notes on Solution form is always included in the report and can be used directly from there.

4.4 Processing a set of ANLDE systems

Web-SynDic can be used for solving a set of test ANLDE systems (ANLDE systems set). An ANLDE systems set is given by a user as a plain text file, or generated automatically by a selected generator.

To start the processing a user should click on the Set of ANLDE Systems link. He/she will be returned with corresponding forms for ANLDE system sets processing, see Fig. 4.

Figure 4: Process a Set of ANLDE Systems

The first form is Load a Set from a Text File. It is used to solve an ANLDE system set stored in a txt-file. The file must be in ANLDE system set format:

- Each ANLDE system is in traditional mathematical text format.
- Comment is started with '#' and follows to the end of a line.
- ANLDE systems are separated by line started with '%'

For example:

```
# It has 2 minimal solutions
x1 + x2 = 2*x1 + 3*x3
x3 + x4 = x1 + 2*x2 + x3
```

```

%
# The same as the previous one, but not in standard ANLDE form.
x2 = x1 + 3*x3
x4 = x1 + 2*x2
%
# It has 40 minimal solutions!
x1 + x2 + x3 = 5*y1 + 2*y2 + y3 + 3*y4

```

To solve the set a user should first of all input file name or press **Browse...** button, then choose the desired file on her/his local machine. He/she can also select an alternative solver to compare solutions with the syntactic algorithm. After that pressing **Solve a set** button starts the processing of the given set.

The second form is **Generate a New Set** and it is used for ANLDE system set generation. To solve generated system set user should choose one of the listed generators and press **Solve a set** button. Alternative solvers can be optionally selected to compare with the syntactic algorithm.

Report on solution provides the following data:

1. Number of ANLDE systems in the set.
2. The set characteristics.
3. Solver's computational metrics on efficiency.
4. Solving machine characteristics.

The **Notes on Solution** form is always included in the report and can be used directly from there.

To save a generated ANLDE system set the **Save a set** button of the second form can be used. The web system opens window containing generated ANLDE system set. Actual saving of the system on a user's local storage can be performed using standard browser features on file saving. Remark that the set is not solved, if a user chooses to save it.

4.5 Web-SynDic Notes

Web-SynDic allows a user to send any her/his opinion to the Web-SynDic system administrator. A special case here is user's disagreement with found solution(s) of the processed ANLDE system.

To compose a general note, i.e. note about Web-SynDic as a whole system, a user clicks on the [Send notes](#) link in Main Menu, whereupon he/she will be returned with the following form.

Figure 5: General note

The user writes as plain text what he/she wish to send, and clicks on the **Send note** button. The composed note will be transferred to the Web-SynDic system administrator by email.

Web-SynDic provides a feature to send note about particular solved ANLDE system(s). When a report on solution is returned, a user can select [Note on solution](#) link for sending more detailed opinion about this solution or just press the **Agree with solution** button whenever he/she satisfied with the solution.

If a user selects [Note on solution](#), then **Notes on Solution** form will appear, see Fig. 6.

With the form the user can choose one of two possibilities:

1. Agreement with the result of solving.
In this case the user may or may not attach ANLDE system(s) to the note, using the **Attach processed system** checkbox.
2. Disagreement with the result of solving.
In this case the processed ANLDE system(s) will always be attached to the note automatically.

Both features are used for backward opinions of Web-SynDic users but the latter is also intended for testing the syntactic algorithms.

WEB-SYNDIC

Process:

- ANLDE System
- Set of ANLDE Systems

Documentation:

- Survey on ANLDE Theory
- User Guide

Send Notes

User Limits

Log In (optional)

Nickname:

Password:

Send Notes on Solution

Dear, guest

Write any opinion about the last ANLDE processing case. For instance:

- what do you think about this ANLDE system or its solution?
- what is your opinion about efficiency of the syntactic algorithm?
- what does the comparison with another solver show?

You can either agree or disagree with the solution. We are glad to meet any disagreement, especially when the solution is incorrect (in any way you mean).

Agree with solution (attach the processed system or set)
 Disagree (the system will be attached automatically)

Your note will be sent to the system administrator (kulakov@cs.karelia.ru).

Any your comment, remark or observation is welcome. We are open for fruitful discussion and criticism.

Petrozavodsk State University, Department of Computer Science
Web-SynDic Team

Figure 6: Note on solution

If a user just presses **Agree with solution** button directly in the report on solution page, a standard message about solution agreement will be send to the Web-SynDic system (for statistics).

4.6 User Limits Management

A user may manage her/his own limits on the solution process.

Changing of user limits can be initialized by clicking [User Limits](#) link in the user menu. Then user limits will appear in content part, see Fig. 7.

A user change limits according with their upper bounds, the latter ones are shown near text fields. Then he/she presses **Submit** button to save the new values. When a user registers in the Web-SynDic system, then values of anonymous user's limits are copied to the profile as the initial limits of the just registered user.

If a user logs in the web system, the changes in limits are saved in user's profile, otherwise they are available during the current session lifetime and lost when the session is over.

Values of upper bounds on user limits are stored may be changed only by the system administrator.

WEB-SYNDIC

Process:

- ANLDE System
- Set of ANLDE Systems

Documentation:

- Survey on ANLDE Theory
- User Guide

Send Notes

User Limits

Log In (optional)

Nickname:

Password:

Department of Computer Science

User Limits

Max. time for solving (sec):	<input type="text" value="15"/>	≤	120
Max. memory for solving (KB):	<input type="text" value="4096"/>	≤	5120
Max. values of coefficients for ANLDE system:	<input type="text" value="10"/>	≤	100
Max. values of coefficients for basis solutions:	<input type="text" value="100"/>	≤	10000
Max. number of equations in ANLDE system:	<input type="text" value="3"/>	≤	10
Max. number of unknowns in ANLDE system:	<input type="text" value="6"/>	≤	15
Max. number of ANLDE systems in a set:	<input type="text" value="3"/>	≤	20
Max. number of basis solutions for ANLDE system:	<input type="text" value="100"/>	≤	100
Max. number of basis solutions in a report:	<input type="text" value="100"/>	≤	100

Petrozavodsk State University, Department of Computer Science
Web-SynDic Team

Figure 7: User limits

5 Sysadmin Manual

System administrator is a privileged user; she/he manages and controls the web system. This includes maintainance of the data store, activity statistics usage and management of users and user limits.

5.1 Logging in as an administrator

To access the administrator privileges you need to log in the Web-SynDic system as administrator. There is predefined nickname *admin* for privileged user in the web system, but you can change it or add a new user with administrator's privileges. Logging is quite typical (see also section 4.2): you enter the nickname and password in the appropriate form and press **Log in** button. If the password is correct and the user with entered nickname has administrator privileges, you will see administrator greeting and some options, which are inaccessible to regular user, see Fig. 8).

If you want to finish your current administrator session and become an anonymous user, just follow [log out](#) link. It is dangerous to leave administrator session opened after you have been finished your work, so **don't forget to log out if you have been logged in as administrator.**

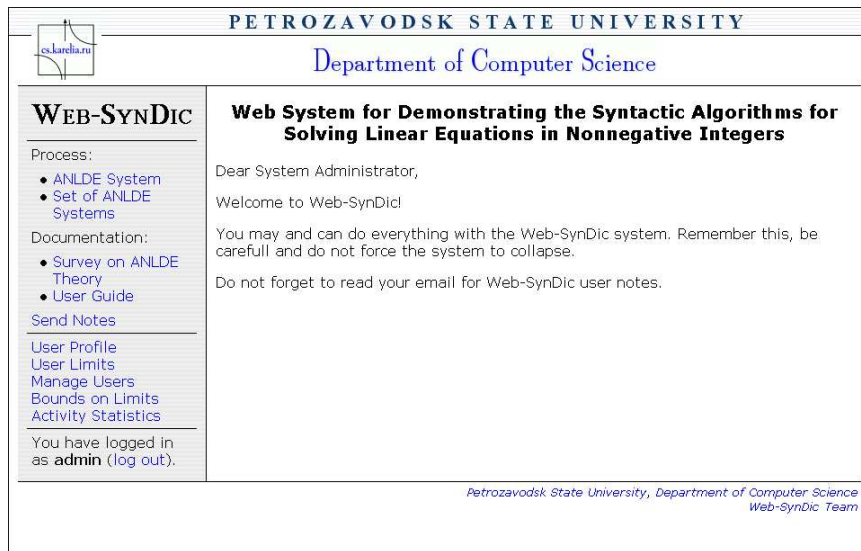


Figure 8: Administrator main window

5.2 User management

One of the specific administrator options is ability to manage other users. The user management means viewing and changing any existing user information and removing user accounts. Follow [Manage Users](#) link to get user management form, see Fig. 9).

To manage a user you need to enter nickname into the form and press **Get Profile** button. If you have administrator privileges and the nickname is valid, you will see the standard form for editing user profile (see section 4.2). If any error has been occurred you get a message with the error description. After editing the information do not forget to press **Submit** button, if you want the changes to be saved.

Be careful while changing password and especially removing accounts, you may easily remove even administrator account.

5.3 Changing bounds on limits

Administrator has ability to change the bounds on limits. Bounds on limits mean a set of constraints which are the upper¹ limits for some parameters needed by a solver or generator, e.g. number of ANLDE systems in a set, maximum time for solving, etc. A regular user can

¹Perhaps in future there will be lower bounds too.

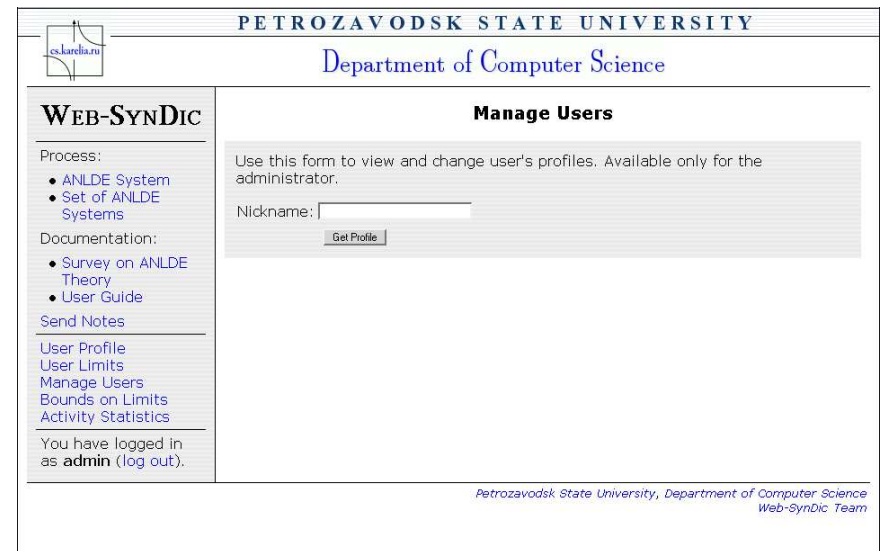


Figure 9: User management

not set her/his user limits (section 4.6) to values greater than the bounds. Please note, that administrator can set her/his own user limits to any values.

To change bounds on limits follow [Bounds on Limits](#) link. If you have administrator privileges you will see form for changing the bounds on user limits and default limits, see Fig. 10). After setting some values do not forget to press **Submit** button.

Be careful while changing bounds on limits, you can overload or even corrupt the web system.

5.4 Viewing activity statistics

Administrator has ability to view activity statistics, collected by the web system. The activity statistics mean summarized information appropriate to selected domain and metrics for current month. Activity statistics domain is a criterion by which statistics information is summarized, e.g. user nickname or IP address. Activity statistics metrics define categories for statistics report, e.g. requests for solving or number of sessions.

To view activity statistics follow the link [Activity Statistics](#). If you have administrator privileges you will see form for viewing activity statistics, see Fig. 11). Here you can choose the type of statistics report.

A statistics report contains time of report generation and statistics information appropriate

to selected domain and metrics for current month. You can choose either nickname and IP address as the domain of the report. Number of sessions, total sessions time, requests for solving, requests for generation, total system time, total work time, agreements with solutions can be selected as metrics for a required report. After pressing **Get Report** button you will see the statistics report. Records in the report are sorted by domain. An example of statistics report with nickname selected as domain and number of sessions selected as metrics is shown in Fig. 12.

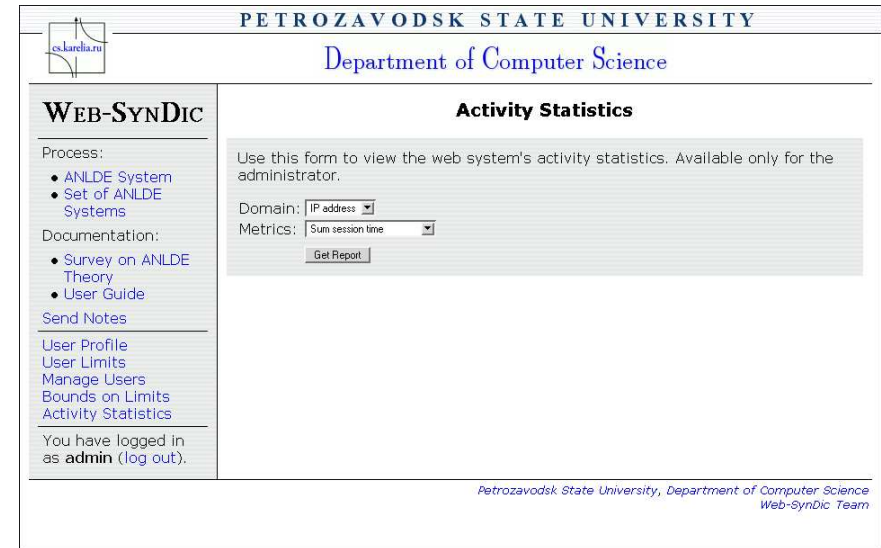


Figure 11: Viewing activity statistics

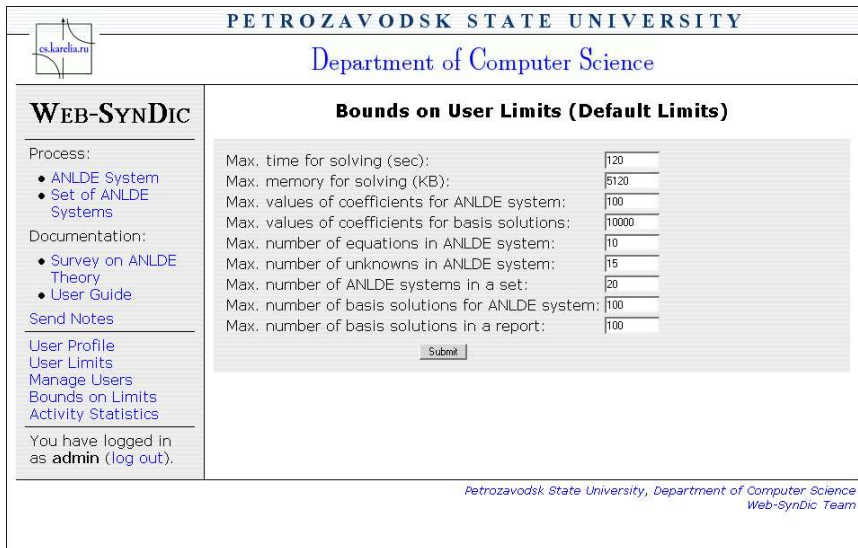


Figure 10: Changing bounds on limits

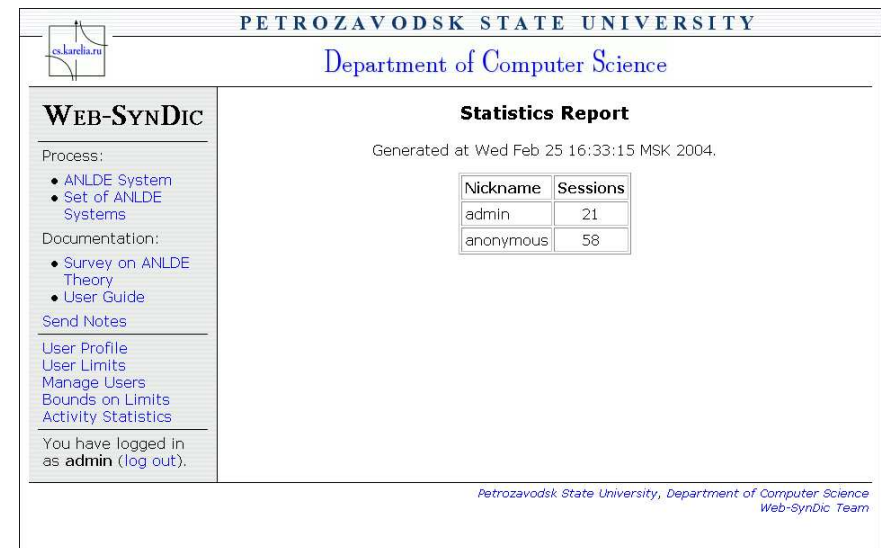


Figure 12: Example of activity statistics report

6 Installation and Configuration Instructions

Software requirements:

Web-SynDic requires Java 1.4 or later (<http://java.sun.com/>) and Jakarta Tomcat 4.1.x (<http://jakarta.apache.org/>).

Web-SynDic distribution:

The Web-SynDic software consists of the platform-independent part and external solving and generation programs. You will need the versions of these programs compiled for your operating system and platform.

Installation instructions:

To install Web-SynDic into Jakarta Tomcat server, copy the Web-SynDic directory or war archive into the Tomcat's webapps directory. Also, you may use the Tomcat Web Application Manager included with the Jakarta Tomcat to install Web-SynDic into the running server. For more details see the Jakarta Tomcat documentation.

Configuration:

Web-SynDic supports the following context parameters:

solver_spooler.path, **generator_spooler.path** : path to the external programs directories.

datastore.path : path to the data store directory, where user accounts and limits are stored.

admin_email : administrator's email for user notes and bug reports.

server.cpu, **server.ram**, **server.os**, **server.nice**, **server.java** : server information to be displayed in the report pages.

You may define the context parameters in the WEB-INF/web.xml file in the following format:

```
<context-param>
<param-name>parameter name</param-name>
<param-value>parameter value</param-value>
</context-param>
```

References

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